Lecture Notes 12

THE NEW INFLATIONARY UNIVERSE

GOOD NEWS: No need to panic. These lecture notes will not be included on the quiz.

INTRODUCTION:

The new inflationary universe is a scenario in which the hot matter of the early universe supercools by many orders of magnitude below the critical temperature of a phase transition predicted by grand unified theories; in the process the universe expands exponentially by many orders of magnitude—hence the name “inflationary.” The word “new” refers to a modification of my original proposal which was suggested independently by Linde and by Albrecht and Steinhardt. They suggested a new mechanism by which the phase transition could take place, solving some crucial problems which were created by my original proposal. The inflationary model is very attractive because it can solve the cosmological problems discussed in Lecture Notes 9 and 11. If correct, it would also mean that grand unified theory mechanisms are responsible for the production of essentially all the matter, energy, and entropy in the observed universe.

SCALAR FIELDS AND THE FALSE VACUUM:

The (original) inflationary universe scenario was developed to solve the magnetic monopole problem, but it quickly became clear that the scenario might solve all three of the problems discussed in Lecture Notes 9 and 11. The scenario contained the basic ingredients necessary to eliminate these problems, but unfortunately the scenario also contained one fatal flaw: the all-important phase transition occurred by the random nucleation of bubbles of the new phase,

very similar to the way that water boils. It was found, however, that this violent boiling would create gross inhomogeneities in the universe. This flaw is completely avoided in a variation known as the new inflationary universe, developed independently by Andre Linde (then at the Lebedev Physical Institute in Moscow, now at Stanford) and by Andreas Albrecht and Paul Steinhardt. (Albrecht and Steinhardt were both at the University of Pennsylvania at the time of their discovery, but Albrecht is now at Imperial College in London.)

In order for the new inflationary scenario to occur, the underlying particle theory must contain a scalar field \( \phi \) which has the following properties:

1. The potential energy function \( V(\phi) \) must have a minimum at a value of \( \phi \) not equal to zero.

2. \( V(\phi) \) must be very flat in the vicinity of \( \phi \approx 0 \). The value \( \phi = 0 \) is usually assumed to be a local maximum of \( V(\phi) \).

3. At high temperature \( T \), the thermal equilibrium value of \( \phi \) should lie at \( \phi = 0 \). (At high temperatures the field will actually fluctuate wildly, but the average value is assumed to be zero. This property can be shown to hold in a wide variety of theories.)

A potential energy function of this general form is shown below:

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \]

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The scalar field $\phi$ that drives the inflation was originally taken to be the Higgs field, but it is now known that this does not work. The Higgs fields are required to have relatively strong interactions in order to induce spontaneous symmetry breaking, which is why the Higgs fields were introduced in the first place. These interactions lead to large quantum fluctuations in the evolution of the field, which in turn leads to unacceptably large inhomogeneities in the mass density of the universe. One must therefore assume the existence of another scalar field, similar to the Higgs field but much more weakly interacting.

So, at high temperatures one expects the scalar field to have a mean value around zero. As the universe expands and cools, the thermal excitations disappear, and the scalar field finds itself in a state of essentially zero temperature, with $\phi \approx 0$. This state is called the false vacuum, and its peculiar properties are the driving force behind the inflationary model.

The false vacuum is clearly unstable, as $\phi$ will not remain forever at a local maximum of $V(\phi)$. However, if $V(\phi)$ is sufficiently flat, then the time that it takes for $\phi$ to move away from $\phi = 0$ can be very long compared to the time scale for the evolution of the early universe. Thus, for these purposes the false vacuum can be considered metastable.

Since the false vacuum has $\phi = 0$ and no other excitations, the mass density has a fixed value which is determined by the potential energy function $V(\phi)$. For a typical grand unified theory, this value can be estimated in terms of the GUT energy scale $E_{\text{GUT}} \approx 10^{14}$ GeV by using dimensional analysis:

$$\rho_f \approx \frac{E_{\text{GUT}}^4}{h^2 c^5} = 2.3 \times 10^{73} \text{ g/cm}^3.$$  \hspace{1cm} (12.1)

The pressure $p$ of the false vacuum is completely determined by the fact that the energy density has a fixed value $\rho_f c^2$. To see this, think of an imaginary piston which is filled with false vacuum and surrounded by ordinary true vacuum, as shown below:

The true vacuum has zero energy density and zero pressure. Suppose now that the piston is pulled out so that the volume of the chamber increases by $\Delta V$. The energy of the system then increases by $\rho_f c^2 \Delta V$, and therefore the agent that moved the piston must have done precisely this amount of work.

$$\Delta W = \rho_f c^2 \Delta V = -p \Delta V$$
Since the pressure on the outside is zero, the agent must be pulling against a negative pressure, which would oppose the motion. Quantitatively, since the work done is $-p\Delta V$, it follows that
\[
p = -\rho c^2.
\] (12.2)

To see the effect of this large, negative pressure, recall the Robertson-Walker equation of motion
\[
\ddot{R} = -\frac{4\pi}{3} G \left( \rho + \frac{3p}{c^2} \right) R.
\] (12.3)

This equation says that both the pressure and the energy density contribute to the retardation of the universal expansion. For the false vacuum, however, $\rho + \frac{3p}{c^2} < 0$, and it follows that $\ddot{R}$ is positive—the false vacuum creates a gravitational repulsion! It is this repulsion which will drive the colossal expansion of the inflationary scenario.

**THE NEW INFLATIONARY UNIVERSE:**

We can now go through the new inflationary scenario step by step. The starting point of a cosmological scenario is, unfortunately, still somewhat of a matter of taste and philosophical prejudice. Some physicists find it plausible to assume that the universe began in some highly symmetrical state. Many others, however, consider it more likely that the universe began in a highly chaotic state, since the number of chaotic configurations is presumably much larger. One advantage of the inflationary scenario, from my point of view, is that it appears to allow a wide variety of starting configurations. It requires only that the initial universe is hot ($kT > 10^{14}$ GeV) in at least some places, and that at least some of these regions are expanding rapidly enough so that they will cool to the critical temperature of the phase transition $T_c$ before gravitational effects reverse the expansion.

In these hot regions, thermal equilibrium would imply $\langle \phi \rangle = 0$, where $\langle \phi \rangle$ denotes the mean value of the field $\phi$ as it undergoes its thermal fluctuations. (Actually, though, the universe has not had time at this point to reach thermal equilibrium. Thus, I need to assume that there are some regions of high energy with $\langle \phi \rangle \approx 0$, and that some of these regions lose energy with $\phi$ being trapped in the false vacuum.) Such a region will cool to $T_c$, and calculations indicate that it will continue to supercool well below $T_c$. The mass density $\rho$ will approach $\rho_1$, the mass density of the false vacuum. To see what happens next, it is easiest to begin by assuming that the region is homogeneous, isotropic, and flat. (Later I will describe what happens when this assumption is dropped.) The region can then be described by the Robertson-Walker flat ($k = 0$) metric
\[
ds^2 = -c^2 dt^2 + R^2(t)dz^2,
\] (12.4)

and the equation of motion becomes
\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G \rho.
\] (12.5)

The solution is given by
\[
R(t) = e^{\chi t},
\] (12.6)

where
\[
\chi = \sqrt{\frac{8\pi}{3} G \rho_1}.
\] (12.7)

This exponential expansion is of course the hallmark of the inflationary model. (For our parameters, $\chi^{-1} \approx 10^{-34}$ sec.) Such a space is called a de Sitter space.

Now let us consider what would happen if the initial region were not homogeneous and isotropic. In that case, one must examine the behavior of perturbations about the Robertson-Walker metric. These perturbations seem to be governed by a “cosmological no-hair theorem”, which states that whenever $p = -\rho c^2 = constant$, then any locally measurable perturbation about the de Sitter metric is damped exponentially on the time scale of $\chi^{-1}$. Any initial particle density is diluted to negligibility, and any initial distortion of the metric is stretched (i.e., redshifted) until it is no longer locally detectable. The theorem has been proven only in the context of linearised approximations, but it is believed by many physicists (Stephen Hawking, myself, and others) to be
valid in all cases. Thus, a smooth de Sitter metric arises naturally, without any need to fine-tune the initial conditions.

(The above paragraphs describe the new inflationary universe with a hot beginning, but there are certainly other possibilities. Alexander Vilenkin (of Tufts University) and Linde have investigated speculative but attractive scenarios in which the universe is created by a quantum tunneling event, starting from a state of absolutely nothing. In these models the universe enters directly into a de Sitter phase. In a similar spirit James Hartle (of the University of California at Santa Barbara) and Stephen Hawking (of Cambridge University) have proposed a unique quantum wave function for the universe, incorporating dynamics which leads to an inflationary era. Linde has also proposed the idea of chaotic inflation, in which inflation is driven by a scalar field which is initially chaotic but far from thermal equilibrium.)

As the space continues to supercool and exponentially expand, the mass density is fixed at \( \rho_t \). Thus, the total energy (i.e., all energy other than gravitational) is increasing! If the inflationary model is right, this false vacuum energy is the source of essentially all the matter, energy, and entropy in the observed universe.

This creation of energy seems to violate our naive notions of energy conservation, but we must remember that there is also an energy associated with the cosmic gravitational field— the field by which everything in the universe is attracting everything else, thereby slowing down the cosmic expansion. Even in Newtonian mechanics one can see that the energy density of a gravitational field is negative. To see this, note that the gravitational field is strengthened as one brings masses together from infinity, but the potential energy of the system is lowered. Thus the stronger field corresponds to a lower energy. In the context of inflation, the energy stored in the gravitational field becomes more and more negative as the universe inflates, while the energy stored in matter becomes more and more positive. The total energy remains constant, and very small— perhaps it is exactly equal to zero.

After the region has undergone exponential expansion for some time, the phase transition must eventually take place. The scalar field is in an unstable configuration, perched at the top of the hill of the potential energy diagram shown on p. 2. It will undergo fluctuations due to thermal and/or quantum effects. Some fluctuations begin to grow, and at some point these fluctuations become large enough so that their subsequent evolution can be described by the classical equations of motion. I will use the term "coherence region" to denote a region within which the scalar field is approximately uniform. The coherence regions are irregular in shape, and their initial size is typically of order \( \chi^{-1} \). Note that \( \chi^{-1} \) is only about \( 10^{-11} \) proton diameters; the entire observed universe will evolve from a region of this size or smaller.

The scalar field \( \phi \) then "rolls" down the potential energy function shown on p. 2, obeying the classical equations of motion derived from general relativity. As long as the spatial variations in \( \phi \) are small, these classical equations take the form

\[
\ddot{\phi} + \frac{3}{R} \dot{R} \phi = - \frac{\partial V}{\partial \phi}.
\]

(12.8)

(The derivation of Eq. (12.8) is a straightforward application of general relativity, but it is beyond the scope of this course.) If the initial fluctuation is small, then the flatness of the potential for \( \phi \approx 0 \) will imply that the rolling begins very slowly. Note that the second term on the left-hand-side of Eq. (12.8) is a damping term, helping to further slow down the speed of rolling. As long as \( \phi \approx 0 \), the mass density \( \rho \) remains about equal to \( \rho_t \), and the exponential expansion continues. The expansion occurs on a time scale \( \chi^{-1} \) which is short compared to the time scale of the rolling. This "slow rollover" is the crucial new feature in the new inflationary universe.

For the scenario to work, it is necessary for the length scale of homogeneity to be stretched from \( \chi^{-1} \) to at least about 10 cm before the scalar field \( \phi \) rolls off the plateau of the potential energy diagram. This corresponds to an expansion factor of about \( 10^{25} \), which requires about 58 time constants \( (\chi^{-1}) \) of expansion. The expected duration of the expansion depends on the precise shape of the scalar field potential, and models have been constructed which yield much more than the minimally required amount of inflation.

When the \( \phi \) field reaches the steep part of the potential, it falls quickly to the bottom and oscillates about the minimum. The time scale of this motion is a typical GUT time of \( \hbar/E_{\text{GUT}} \approx 7 \times 10^{-39} \) sec, which is very fast compared to the expansion rate. The scalar field oscillations are then quickly damped by the couplings to the other fields, and the energy is rapidly converted into a thermal equilibrium mixture of particles. (From a particle point of view, the scalar field oscillations correspond to a state of spinless particles, just as an oscillating
electromagnetic field corresponds to a state of photons. The damping of the scalar field is just the field theory description of the decay of these particles into other kinds of particles.) The release of this energy (which is just the latent heat of the phase transition) reheats the region back to a temperature of order \( kT \approx 10^{14} \) GeV.

From here on the standard scenario takes over. The era of inflation has set up precisely the initial conditions that had previously been assumed in standard cosmology. The length scale of homogeneity increases to a value greater than \( 10^{10} \) light-years (and perhaps much greater than \( 10^{10} \) light-years) by the time \( T \) falls to 2.7 K.

**SOLUTIONS TO THE COSMOLOGICAL PROBLEMS:**

Let me now explain how the three problems of the standard cosmological scenario discussed in Lecture Notes 9 and 11 are avoided in the inflationary scenario. First, let us consider the horizon/homogeneity problem. The problem is clearly avoided in this scenario, since the entire observed universe evolves from a single coherence region. This region had a size of order \( \chi^{-1} \) at the time when the fluctuation began to grow classically. This size is much smaller than the sizes that are relevant in the standard model at these times, and the region therefore had plenty of time to come to a uniform temperature before the onset of inflation. The exponential expansion causes this very small region of homogeneity to grow to be large enough to encompass the observed universe.

The flatness problem is avoided by the dynamics of the exponential expansion of the coherence region. As \( \phi \) begins to roll very slowly down the potential, the evolution of the metric is governed by the mass density \( \rho_t \). Assuming that the coherence region (or a small piece of it) can be approximated by a Robertson-Walker metric, then the scale factor evolves according to the standard equation:

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G \rho - \frac{k c^2}{R^2},
\]  

where \( k = +1, -1, \) or 0 depending on whether the region approximates a closed, open, or flat universe, respectively. (There could also be perturbations, but the cosmological no-hair theorem guarantees that they would die out quickly.) In this language, the flatness problem is the problem of understanding why the \( kc^2/R^2 \) term on the right-hand-side is so extraordinarily small. But as the coherence region expands exponentially, the mass density \( \rho \) remains very nearly constant at \( \rho_t \), while the \( kc^2/R^2 \) term is suppressed by at least a factor of \( 10^{50} \). This provides a "natural" explanation of why the value of the \( kc^2/R^2 \) term immediately after the phase transition is smaller than that of the other terms by a tremendous factor.

Except for a very narrow range of parameters, this suppression of the curvature term will vastly exceed what required by present observations. This leads to the prediction that the \( kc^2/R^2 \) term of Eq. (12.9) should remain totally negligible until the present era, and even far into the future. This implies that the value of \( \Omega \) today is expected to be equal to one with a high degree of accuracy.

At present the status of the prediction that \( \Omega = 1 \) is controversial. Certainly it is not favored by most of the observations, which typically indicate a value in the range of 0.2 - 0.4. However, a reliable determination of \( \Omega \) is exceedingly difficult. It turns out the most of the mass in the universe is in the form of unidentified "dark matter"— matter whose existence is inferred only through its gravitational effects. In the absence of any knowledge about the nature of this dark matter, it is very difficult to say how much of it there is. Furthermore, there are two lines of argument that strongly suggest a value of \( \Omega \) near one. First, there is evidence that galaxies over large regions of space have bulk velocities that differ significantly from the Hubble flow. These perturbations are believed to be caused by gravity, and it is difficult to explain the magnitude of the effect if \( \Omega \) is not near one. Second, the extreme uniformity of the cosmic background radiation has to be reconciled with the inhomogeneity of the present universe. One understands that small nonuniformities in the early universe are amplified by gravitational instabilities— that is, a region with a slight excess mass density will create a slightly stronger than average gravitational field, pulling in more mass, which creates a still stronger gravitational field, etc. The extent of this effect, however, depends on the mass density, and it is difficult to explain how such a clumpy universe can develop from such a smooth beginning if \( \Omega \) is small. Thus, most investigators agree that \( \Omega = 1 \) is not ruled out. However, the viability of the inflationary model in future years will depend critically on whether the prediction for the mass density of the universe remains acceptable.
The prediction for the mass density is complicated further by another question—the possibility of a nonzero cosmological constant. Historically, the cosmological constant rests on a shaky foundation—it was first introduced by Einstein in an attempt to salvage a static model of the universe. It corresponds to assigning a nonzero mass density \( \rho_v \) to the vacuum. This fixed vacuum mass density (with a corresponding negative vacuum pressure) creates a gravitational repulsion exactly like the false vacuum discussed above, and Einstein wanted to use it to stabilize his model universe against gravitational collapse. Einstein abandoned the cosmological constant when it was discovered that the universe is expanding, but the existence of a nonzero cosmological constant remains a possibility. From the perspective of present-day particle physics, however, the possibility of a significant cosmological constant seems unlikely. The vacuum of modern particle theories is a very complicated state, so particle theorists find it difficult to understand why the cosmological constant is not huge. There must therefore be some mechanism of suppression that is not understood, and it is generally supposed that such a mechanism would suppress the cosmological constant either to zero, or to a value that is cosmologically irrelevant.* In any case, if the cosmological constant is nonzero, then the value of the corresponding vacuum mass density \( \rho_v \) must be counted in the prediction of inflation:

\[
\rho_{\text{matter}} + \rho_v = \rho_c .
\]

(For those familiar with Einstein’s original definition of the cosmological constant \( \Lambda \), the corresponding mass density is given by \( \rho_v = \Lambda / 8 \pi G \).)

Finally, we turn to the monopole problem. Recall that in the standard scenario, the tremendous excess of monopoles was produced by the disorder in the Higgs field (i.e., by the Kibble mechanism). In some versions of the new inflationary model, the Higgs field and the scalar field \( \phi \) which drives the inflation are one and the same. In such cases the Higgs field is approximately uniform throughout the coherence region, which has been stretched by inflation to be much larger than the observed universe—the Kibble mechanism thereby produces less than one monopole in the observed universe. (Some monopoles would still be produced by thermal fluctuations after reheating, but this number would be negligible in most particle theories.) In many recent versions of the inflationary model, \( \phi \) and the Higgs field are distinct. In such cases one must arrange for the Higgs field to acquire its nonzero expectation value either before or during the inflationary era, so that the monopole density is diluted by the inflation.

CONCLUSION:

In conclusion, inflation must be considered a speculative theory. Nonetheless, I believe that the basic idea of inflation—the idea that the universe went through a period during which it expanded exponentially while trapped in a false vacuum—is probably correct. It is a very simple and natural idea in the context of spontaneously broken gauge theories, and it seems to solve some very fundamental cosmological problems.

An important feature of the inflationary model is the fact that it makes testable predictions. The predictions are unfortunately very difficult to test, but the tests still might succeed within the foreseeable future. The simplest and most unavoidable prediction is the one discussed above—the prediction of the mass density of the universe. In addition, inflation makes a testable prediction for the spectrum of primordial mass density inhomogeneities of the universe. These inhomogeneities arise in the inflationary model from quantum fluctuations of the scalar field which drives inflation. While the calculation depends slightly on the unknown form of the energy density function for the scalar field, it is nonetheless true that almost all inflationary models yield a spectrum that is near to the scale invariant form known as the Harrison-Zeldovich spectrum. It is plausible that these inhomogeneities are responsible for the formation of galaxies, and progress is being made in testing whether the observed distribution of galaxies can evolve from initially scale-invariant perturbations. In addition, the density inhomogeneities can also be seen directly in the cosmic background radiation, and the COBE satellite has now given us the first measurements of this effect. The graph on the opposite page shows the results from COBE for the two-point correlation function,

\[
C(\alpha) = \langle T_1 T_2 \rangle - \langle T \rangle^2 .
\]  

(12.10)
Here $T_1$ and $T_2$ refer to the temperature of two points on the sky that are separated by an angle $\alpha$. The brackets indicate to average over all points measured. The shaded region on the graph shows the prediction of a scale-invariant spectrum, and the agreement between observation and theory looks excellent. (The thickness of the shaded region reflects the intrinsic quantum uncertainties—the theory predicts only a probability distribution for the inhomogeneities, and not a precise pattern. The shaded region was drawn at the 68% confidence level, which means that 68% of the true data points are predicted to lie within the shaded region.)

Although I think inflation is basically right, I want to emphasize that we clearly do not yet have the details straight. In order to compute the details of the quantum fluctuations which take place during the phase transition and perhaps lay the seeds for galaxy formation, we must first understand the details of particle physics at GUT energy scales. To compare the predictions of inflation with present-day observations, we must also have a detailed understanding of the evolution of the universe from $10^{-30}\sec$ to the present, which at the least would require an understanding of the nature of the dark matter. We are presumably some distance from any of these goals. Thus, I expect that the interface between particle physics and cosmology will remain an exciting area of research for some time to come.